Written Exam for the M.Sc. in Economics, Winter 2020-21

Advanced Macroeconomics: Structural Vector Autoregressive Analysis: Solution

About the exam

The topic for this project examination is a small open economy growth model. The purpose of the examination is to assess your understanding of structural vector autoregressive (VAR) models. Substantial emphasis will be placed on using your programming skills in MATLAB. Specifically, the examination assesses theoretical and practical knowledge of structural vector autoregressive models within stationary and non-stationary frameworks including assessing empirical results, using different approaches to identify VAR models and be able to use MATLAB to generate empirical results. You can use any MATLAB functions that you have programmed yourself or any function uploaded to Absalon during the course except when otherwise stated. You are not allowed to use other programs or built-in MATLAB functions except for those that are specified in the questions below. The assignment requires some additional coding.

Most questions in the examination are applied, concerning the empirical example outlined below. When you answer these empirical questions, please explain and motivate your answers as detailed as possible, preferably with reference to the underlying theory.

This exam focuses on a small open economy growth model. The steady-state solution to this particular model can be summarized by the following two equilibrium relations

$$\ln Y_t - \ln I_t = v_1$$

where Y_t is GDP, I_t is investments and v_1 is a constant, and

$$\ln Y_t - \ln C_t + (1-b)\ln P_t = v_2$$

where C_t is total consumption, b is the share of consumption of domestically produced goods in total consumption, P_t is the terms-of-trade and v_2 is a constant.

Assume that $x_t = \begin{bmatrix} \ln P_t & \ln Y_t & \ln C_t & \ln I_t \end{bmatrix}' = \begin{bmatrix} p_t & y_t & c_t & i_t \end{bmatrix}'$ and suppose that this vector is integrated of order one and ordered as in x_t we find that the theoretical cointegration vector β is given by

$$\beta' = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 1-b & 1 & -1 & 0 \end{bmatrix}$$

The assignment will guide you through an empirical analysis of the time series vector stated above including estimation and analysis of the cointegrated VAR model, identification of the structural cointegrated VAR model and robustness analysis.

Regarding the data for the exam paper, please note the following:

- All assignments are based on different data sets. You should use the data set (monthly data covering the period 1995:01-2015:01) located in the MATLAB file 1234.mat, where 1234 is your exam number. This MATLAB file contains the data (y), the dates (dates) and the name of the variables (names). You can load this file into MATLAB directly using 'load 1234.mat'. In case you cannot find your exam number, you can use the 1000.mat file.

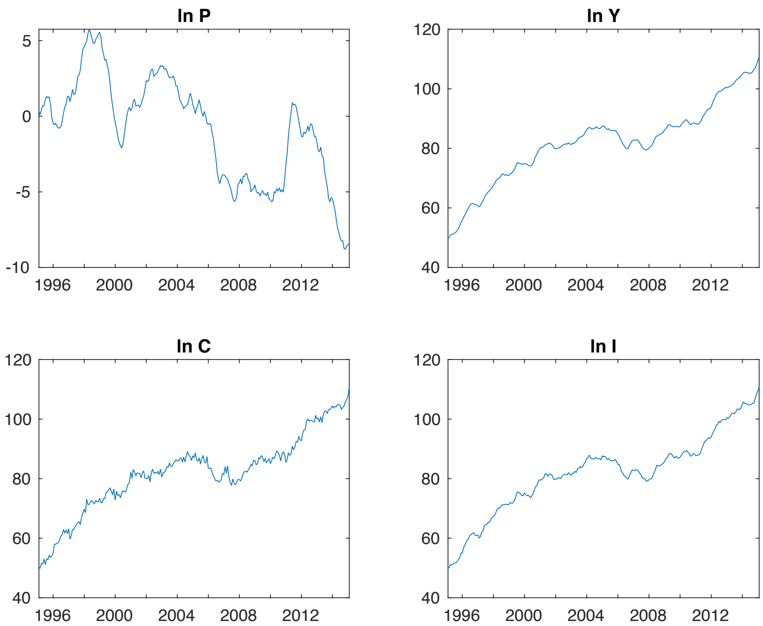
- To avoid that some data sets are more difficult to handle than others, the data sets are artificial (simulated from a known data generating process), and they behave, as close as possible, like actual data.

The proposed solution below is based on the data set 1000.mat

1. The data is already in natural logarithms (real money balance and real GDP are in logs whereas the money market rate is in percent). Plot the data and perform graphical analysis in order to assess the degree of integration of all four variables.

Answer:

Figure 1: Plot of data.



It is clear from this plot that there are linear trends in the data. Output $\ln Y$, consumption $\ln C$ and investment $\ln I$ are all increasing over time. They also seem to follow each other in downturns and upturns. Terms of trade $\ln P$ on the other hand has a downward trend and could be stationary around a linear trend. The other three variables appear non-stationary. The similar behavior of output, consumption and investment could indicate that these three variables are cointegrated. The question is whether we need to allow for a linear trend in the cointegration vector. This will be tested below.

The Vector Error Correction Model

Suppose that the four variables in x_t are either $I(1)$ or $I(0)$ and that the underlying data generating process is a Vector Autoregressive (VAR) model,

$$x_t = \nu + A_1 x_{t-1} + \ldots + A_p x_{t-p} + u_t \tag{1}$$

where x_t is defined above, p is the lag length, ν is a constant term and u_t is a vector of zero mean white noise process with covariance matrix Σ_u such that $u_t \sim (0, \Sigma_u)$. Then we can rewrite the VAR model as the following Vector Error Correction (VEC) model

$$\Delta x_t = \nu + \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + \ldots + \Gamma_{p-1} \Delta x_{t-p+1} + u_t \tag{2}$$

where

$$\Pi = -\left(I_4 - A_1 - \ldots - A_p\right)$$

and

$$\Gamma_i = -\left(A_{i+1} + \cdots + A_p\right) \quad \text{for } i = 1, \ldots, p-1.$$

The rank of Π is equal to the number of cointegration vectors r and Π can be decomposed as a product of two $4 \times r$ matrices of full rank, $\Pi = \alpha\beta'$ where α is the $4 \times r$ adjustment coefficients and β is the $4 \times r$ cointegration vectors.

2. Formulate a well-specified VEC model for x_t similar to the VEC model above. Explain your workflow and how you argue for your choice of the number of autoregressive lags in the VEC model (and in its associate VAR model).

   Answer:

   There are several different approaches that can be used to determine the number of lags in the underlying VAR model. All approaches are based on estimates of a VAR in levels with a constant term. Three approaches have been used during the course: lag length determination using information criteria, general-to-specific and specific-to-general sequences. It is irrelevant which one is used here. All approaches should lead to the same lag length.

   Here we will apply information criteria (Akaike, Schwarz and Hannan-Quinn) and choose the lag length that minimizes these measures. The workflow should start with a maximum lag length and then we compute these criteria for each lag length $p = 1, \ldots, p_{\max}$ using the same number of observations for each lag length. The function pfind.m produces the following output assuming that $p_{\max} = 12$. From this table (and using all three criteria) we learn that the optimal lag length is 2. The same result holds for all data sets.

p	SIC	HQC	AIC
1	-5.7007	-5.8796	-6.0005
2	-6.4541	-6.7761	-6.9939
3	-6.1470	-6.6121	-6.9267
4	-5.8384	-6.4467	-6.8580
5	-5.5581	-6.3095	-6.8176
6	-5.2490	-6.1435	-6.7485
7	-4.9099	-5.9475	-6.6492
8	-4.6008	-5.7816	-6.5801
9	-4.2733	-5.5972	-6.4925
10	-4.0096	-5.4766	-6.4687
11	-3.6968	-5.3070	-6.3958
12	-3.4100	-5.1633	-6.3489

Alternatives to using information criteria as outlined above is to use one of the following approaches:

- Top-down sequential testing (general-to-specific): The VAR(p) model is

$$y_t = \nu + A_1 y_{t-1} + A_2 y_{t-2} + \ldots + A_p y_{t-p} + u_t$$

  where $\nu = A_0$. Start with a maximum number of lags p_{max} testing a sequence of null hypotheses: $\mathbb{H}_0$: $A_{p_{\max}} = 0$ vs. $\mathbb{H}_1$: $A_{p_{\max}} \neq 0$, $\mathbb{H}_0$: $A_{p_{\max-1}} = 0$ vs. $\mathbb{H}_1$: $A_{p_{\max-1}} \neq 0$, ..., $\mathbb{H}_0$: $A_1 = 0$ vs. $\mathbb{H}_1$: $A_1 \neq 0$. Process terminates when there is a rejection. Use Wald or LR tests.

  Using this approach and assuming that the maximum lag length is 4, we find the following result also suggesting 2 lags in the VAR model.

Lag	Log Likelihood	LR stat	p-value
1	-620.98	2581.35	0.000
2	-484.36	273.26	0.000
3	-475.37	17.97	0.325
4	-467.61	15.51	0.488

- Bottom-up sequential testing (specific-to-general): Reverse the procedure above, start with $p_{\min}$ testing for autocorrelation in the residuals (using for example a multivariate test). Add lags until there is no significant autocorrelation. Applying this approach we still find 2 lags in the VAR model.

Answers using either of these two latter approaches should also be accepted and receive full points if correctly implemented and explained.

3. Test for multivariate autocorrelation, heteroscedasticity and normality. Does your model satisfy the underlying assumptions? If the multivariate tests of autocorrelation and ARCH reject the null hypotheses, apply univariate tests for autocorrelation and ARCH in the residuals in each equation. You are allowed to use the built-in MATLAB functions lbqtest and archtest.

   Answer: To verify that the model is well-specified when assuming that the lag length is equal to 2, we next test for autocorrelation and heteroscedasticity in the residuals and the null hypothesis that the residuals are normally distributed. We should use multivariate tests provided in the functions: portman.m, march.m and multnorm.m. First we need to re-estimate the VAR(2) model using VARls.m. We allow for a constant term but no linear trend. The argument used to exclude a linear trend is that the VAR in levels can be re-written as a VAR in first differences (and as a VEC model) by subtracting y_{t-1} from both sides of the levels VAR leaving the constant term and the residuals unaffected. We obtain the following results:

   - Portmanteau test.

Test	Statistic
Tested order:	6
Test statistic	67.749
p-value	0.35055
Adjusted test statistic	68.953
p-value	0.31364
degrees of freedom	64

We cannot reject the null hypothesis that there is no autocorrelation present in the residuals.

- Tests for Multivariate ARCH.

Test	Doornik-Hendry
test statistic:	426.48
p-value	0.17369
degrees of freedom	400

We cannot reject the null hypothesis that there are no ARCH-effects in the residuals.

- Tests for non-normality.

Test	Doornik-Hansen	Lütkepohl
joint test statistic:	2.7898	2.7056
p-value	0.94685	0.95145
degrees of freedom	8	8
Skewness only	0.74727	1.2342
p-value	0.94537	0.87244
kurtosis only	2.0425	1.4714
p-value	0.72794	0.83169

We find that we cannot reject the null hypothesis that the residuals are normally distributed.

Note that results may differ depending on the tested orders used when testing for autocorrelation and ARCH. However, the results should all point in the direction of 2 lags in the VAR model. It seems as if this is the optimal lag length in the present setting. This conclusion holds for all data sets. There is no need for using the univariate tests, i.e., testing residuals in each equation.

4. For your preferred model, proceed by testing for cointegration using the MATLAB function jcitest. Explain your approach and how you find the number of cointegration vectors, that is, the rank r, in the system allowing for (i) a constant term in the cointegration vector and (ii) both a constant and a linear trend in the cointegration vector. Do you use different sources of information when determining the rank? If so, explain how you arrive at your decision.

   Answer: Starting with the hypothesis that $r = 0$ we find that the null hypothesis that $r = 0$ is rejected at the 1 percent level. Increasing the rank we find that we can reject the null that $r = 1$. Turning to the null that $r = 2$ we find that we cannot reject this hypothesis. The data, therefore, suggest the presence of2 cointegration vectors.

We can also use the estimated eigenvalues as an alternative source of information when determining the rank. As is clearly illustrated in the table above, the eigenvalue falls substantially when increasing the rank from 2 to 3. Our conclusion that the rank is equal to 21 is clearly supported.

A good answer must include a description of the approach, the so called Pantula principle, where we start by considering the null hypothesis that the rank is 0 and then we test the null that the rank is 1 and so on. A sensitivity analysis of the unimportance of the number of lags could be included and, if so, it should be given additional credit.

Under the assumption that there is a constant term in the cointegratiopn vector and using the command [h,pValue,stat,cValue,mles] = jcitest(y,'lags',p-1,'model','H1'); we find the following result. Note that we need to tell matlab that the lag length in the VEC model is equal to $p - 1$, not p.

r	h	stat	cValue	pValue	eigVal
0	1	176.4483	47.8564	0.0010	0.3485
1	1	74.0441	29.7976	0.0010	0.2426
2	0	7.6354	15.4948	0.5493	0.0313
3	0	0.0337	3.8415	0.8547	0.0001

The estimated cointegration vectors (assuming that the rank = 2) are $[\ 0.1293 \quad 1.0000 \quad -0.5519 \quad -0.44$
and $[\ 0.0133 \quad 1.0000 \quad 0.0460 \quad -1.0461\ ]$

When allowing for a linear trend in the cointegration vector we find (using the command [h,pValue,stat,cValue,mles] = jcitest(y,'lags',p-1,'model','H*');)

r	h	stat	cValue	pValue	eigVal
0	1	183.5221	63.8766	0.0010	0.3540
1	1	79.1017	42.9154	0.0010	0.2501
2	0	10.3213	25.8723	0.9106	0.0313
3	0	2.7130	12.5174	0.9087	0.0113

The test results above strongly suggest that there are 2 cointegration vectors present in our system. Estimated cointegration vectors seem to mirror the theoretical cointegration vector.

5. Perform a test of the null hypothesis that there is no linear trend in the cointegration vectors.

Answer: To test whether the linear trend in the cointegration vector is equal to zero, we use the following code:

```
RR = [ 0  0  0  0  1 ]';
[h0,pValue2,stat,cvalue2,mles1] = jcontest(y,2,'Bcon',RR,'lags',p-1,'model','H');
display(mles1.paramVals.B,'Estimated cointegration vector');
display(mles1.paramVals.d0,'Estimated linear trend in cointegration vector');
display(pValue2,'Testing presence of linear trend in beta');
```

Implementing this code we find that we cannot reject the null hypothesis that the coefficient associated with a linear trend in the cointegration vector is equal to zero. The estimated cointegration vector is

$$\begin{bmatrix} -0.3717 & -0.0465 \\ -2.8752 & -3.4932 \\ 1.5868 & -0.1608 \\ 1.2901 & 3.6541 \end{bmatrix}$$

and we need to confirm that the estimated linear trend in cointegration vector is zero. The p-value of the LR test is 0.1115 suggesting a non-rejection of the null hypothesis.

6. Impose your preferred rank and the preferred specification of the deterministic component in the cointegration vector found in the previous question and test hypotheses on the cointegration space using the MATLAB function jcontest. Test for exclusion, stationarity and weak exogeneity. Explain the meanings of these tests.

   Answer: To implement these tests we use the following code:

```
%[h7] = jcontest(y,2,'Bvec',[1 0 0 0]',[0 1 0 0]',[0 0 1 0]',[0 0 0 1]','lags',p-1,'model','H1')
%%
% Exclusion
[h2,pValue2] = jcontest(y,2,'Bcon',[1 0 0 0]',[0 1 0 0]',[0 0 1 0]',[0 0 0 1]','lags',p-1,'model','H1')
%%
% Weak exogeneity
[h1,pValue1] = jcontest(y,2,'Acon',[1 0 0 0]',[0 1 0 0]',[0 0 1 0]',[0 0 0 1]','lags',p-1,'model','H1')
```

   Using this code we find that we always reject stationarity and exclusion whereas we cannot reject the null that the adjustment coefficients are equal to zero in the first two equations but reject the null that they are equal to zero in the last two equations. These results concerning weak exogeneity can vary across data sets but stationarity and exclusion test results are uniform across all data sets.

7. Interpret the unrestricted estimated cointegration vectors in light of the theoretical model above. Do you find plausible values of the parameters in the estimated cointegration vectors?

   Answer: The estimated cointegration vectors assuming that the rank = 2 and that there is only a constant term in the cointegration vector are $[\ 0.1293 \quad 1.0000 \quad -0.5519 \quad -0.4487\ ]$ and $[\ 0.0133 \quad 1.0000 \quad 0.0460 \quad -1.0461\ ]$. We can then compare these estimates with the theoretical cointegration vector

$$\beta' = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 1-b & 1 & -1 & 0 \end{bmatrix}$$

   Comparing these we note that the second estimated cointegration vector is close to the first theoretical vector. The first and third coefficients are close to zero whereas the remaining parameters are close to 1 but with opposite signs. The first estimated vector is different from the second theoretical vector. It is not clear whether the fourth parameter is different from zero as it should be according to the theoretical vector.

8. Test formally whether the theoretical cointegration vectors are in line with the information in the data using the MATLAB function jcontest. Estimate the parameter b in the theoretical cointegration vector. Do you obtain a plausible value of b? Explain how your tests relate to the exclusion and stationarity tests.

   Answer: First we consider tests of the theoretical cointegration vectors. The problem is how to handle the constant b present in the vector. We know from the theoretical model that $0 < b < 1$ since it is a share of consumption of domestically produced goods in total consumption. In a closed economy $b = 0$ and therefore the theoretical cointegration vector is

   $$\beta' = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 1 & -1 & 0 \end{bmatrix}$$

   We can test whether these are in the data using the following codes:

```
% Testing first theoretical cointegration vector allowing the second vector
to be freely estimated
RR = [ 0 1 0 -1]';
[h0,pValue2,stat,cvalue2,mles1] = jcontest(y,2,'Bvec',RR,'lags',p-1);
display(mles1.paramVals.B,'Estimated cointegration vector');
display(pValue2,'Testing hypotheses on beta');
```

   We obtain the following estimated cointegration vector

   $$\begin{matrix} 0 & 0.3685 \\ 1.0000 & 0.7968 \\ 0 & -1.5952 \\ -1.0000 & 0.7968 \end{matrix}$$

   and the p-value $= 0.0646$ implying that we cannot reject this hypothesis at the 10 percent level.

   Testing whether the second cointegration vector is present in the data we use the following code

```
% Testing second theoretical cointegration vector allowing the second vector
to be freely estimated
RR = [ 1 1 -1 0]';
[h0,pValue2,stat,cvalue2,mles1] = jcontest(y,2,'Bvec',RR,'lags',p-1);
display(mles1.paramVals.B,'Estimated cointegration vector');
display(pValue2,'Testing hypotheses on beta');
```

   We obtain the following estimated cointegration vector

   $$\begin{matrix} 1 & -1.3047 \\ 1 & 1.3789 \\ -1 & 0.0742 \\ 0 & -1.4553 \end{matrix}$$

and the p-value = 0.000 implying a clear rejection of this vector.

Then we turn to the parameter b. One way to look at this is to use different values of b to find a value where the theoretical vector is not rejected. Another approach is to implement a restriction on the cointegration space allowing b to be estimated. In matlab, given it's restrictions on how to impose restrictions, the former approach is easier to implement. In this particular exercise we know that the data is generated using a particular value of b also suggesting that the former approach could be used.

Using a sequential approach, we let b to take on values from 1 down to 0.5 in steps of 0.05 to find the value of b where the p-value of a test of the second theoretical vector cannot be rejected. Using the

```
% Testing second theoretical cointegration vector allowing the second vector
to be freely estimated
b = 0.75; RR = [ 1-b 1 -1 0]';
[h0,pValue2,stat,cvalue2,mles1] = jcontest(y,2,'Bvec',RR,'lags',p-1);
display(mles1.paramVals.B,'Estimated cointegration vector');
display(pValue2,'Testing hypotheses on beta');
```

we find that this cointegration vector cannot be rejected (the p-value is 0.1730) and the implied value of $b = 0.75$. A good answer must include a discussion about how b can be inferred and an attempt to find a value of b. The theoretical value is 0.75 for all data sets.

9. Plot both the unrestricted and theoretical cointegration vectors. Interpret your results.

   Answer: Using the value $b = 0.75$ to compute the theoretical cointegration vectors and the unrestricted estimate found when testing for cointegration above we obtain the following graph.

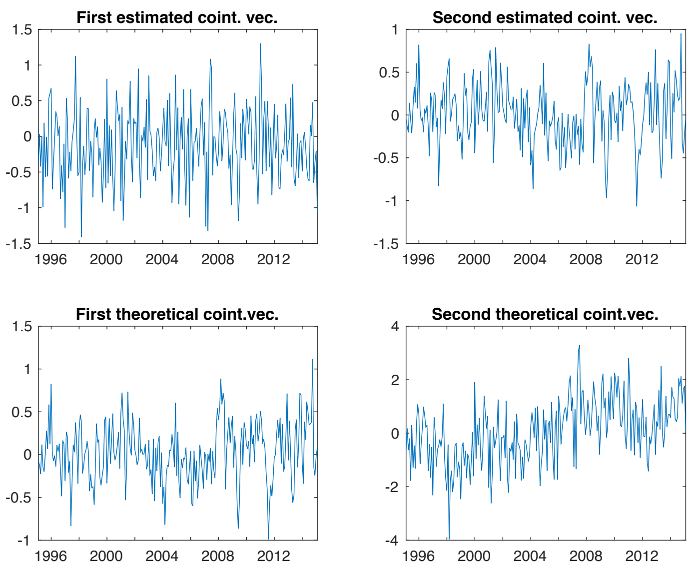


Figure 2:

10. Split the sample into two equal sized sub-samples and perform tests for cointegration using the MATLAB function jcitest. Comment on the importance of the sample length for these tests.

    Answer: We split the sample in two equal parts, observations 1 to 120 in the first sub-sample and observations 121 to 241 in the second sub-sample.

    Testing for cointegration using the first sub-sample we obtain

r	h	stat	cValue	pValue	eigVal
0	1	100.0973	47.8564	0.0010	0.3546
1	1	48.4197	29.7976	0.0010	0.2801
2	0	9.6365	15.4948	0.3528	0.0561
3	0	2.8267	3.8415	0.0927	0.0237

    and for the second sub-sample we obtain

r	h	stat	cValue	pValue	eigVal
0	1	106.6691	47.8564	0.0010	0.4187
1	1	42.1205	29.7976	0.0014	0.2549
2	0	7.1087	15.4948	0.6010	0.0390
3	0	2.3795	3.8415	0.1232	0.0198

    As can be seen in these tables the main conclusion drawn when using the full sample still holds. Point estimates do change but given a well-behaved data such as the present one, results are unchanged. But, using actual data we expect to find differences, p-values tend to increase when using shorter time series illustrating that tests for cointegration and unit roots are biased when using short samples.

Identification of Structural Model

11. Impose $r = 2$ and the theoretical cointegration vector and re-estimate the VEC model using the full sample and using your preferred lag length found above. Suggest an identification scheme including names of the four structural shocks in the VAR/VEC system using long-run identification. If you cannot provide names for these shocks, try to explain how they affect the data under the maintained assumptions.

    Answer: In the present VAR model we have four variables and thus four structural shocks. In addition, we have established that there are two cointegration vectors in the system. Therefore, there are two common trends or permanent shocks and two transitory shocks. Given that we have assumed a small open economy it seems natural to assume that we have one foreign trend and one domestic trend. Furthermore, a small open economy assumption suggests that the domestic trend cannot affect the foreign variable (terms-of-trade) in the long-run. The transitory shocks are both domestic and relates to consumption and investment demand changes (or shocks). These transitory shocks have no long-run effect on any variable.

12. Write down the reduced form and structural form Common Trends model consistent with the VEC model. Show how these two representations are related. What is the

consequence for the long-run multiplier if we assume that the rank $r = 2$?

Answer: The VEC model stated in this problem set can then be rewritten as a Common Trends model

$$x_t = \Xi \sum_{i=1}^{t} u_t + \Xi^*(L)u_t + y_0^*$$

where

$$\Xi = \beta_\perp \left[\alpha'_\perp \left(I_K - \sum_{i=1}^{p-1} \Gamma_i \right) \beta_\perp \right]^{-1} \alpha'_\perp$$

and where $\alpha_\perp$ and $\beta_\perp$ are orthogonal complements to α and β respectively.

Imposing an identification scheme on the reduced form model above allow us to formulate the following structural Common Trends model

$$x_t = \Xi \sum_{i=1}^{t} B_0^{-1} w_i + \Xi^*(L)B_0^{-1}w_t + y_0^*$$

$$x_t = \underbrace{\Xi B_0^{-1}}_{\Upsilon} \sum_{i=1}^{t} w_i + \Xi^*(L)B_0^{-1}w_t + y_0^* = \Upsilon \sum_{i=1}^{t} w_i + \Xi^*(L)B_0^{-1}w_t + y_0^*$$

where Υ is the matrix of long-run multipliers, it measures the long-run effect of the common trends (or the permanent shocks). In our case with two transitory shocks, we know that the last two columns of Υ are equal to zero (given that we order permanent shocks first in the vector of structural shocks). The small open economy assumption implies a zero restriction in the first or second column of Υ depending on how we order the variables. Other parameters in Υ are freely estimated. Also note that the long-run effects of the stationary part $\Xi^*(L)B_0^{-1}w_t$ goes to zero as $j \to \infty$. The rank of Υ is the same as the rank of Ξ, i.e., rank $K - r$.

The interpretation of the permanent and transitory shocks in the current setting is discussed above. We associate the two permanent shocks with a foreign and a domestic trend. These shocks have permanent effects on at least one of the four variables in our system. To just identify these two shocks we need to introduce one restriction. We then have two transitory shocks, the consumption and investment demand shocks. These shocks have only short-term effects on the four variables. To identify the two transitory shocks we need to impose one restriction on B_0^{-1}.

A good answer must include a discussion of possible shocks affecting the VAR/VEC system. The arguments above may not be the only available option, but it is essential that the answer includes a motivation and main arguments based on economic model or intuition.

13. Outline how the MATLAB solver can be used to impose long-run restrictions in your model.

    Answer: We have mentioned above that we need to impose one restriction on Υ to identify the two permanent shocks and one restriction on B_0^{-1} to identify the transitory shocks. Given the ordering of the time series vector state dabove, i.e., $x_t =$

$\begin{bmatrix} p_t & y_t & c_t & i_t \end{bmatrix}'$ we impose the restriction that the first element in the second column of Υ is equal to zero. This restriction implies that the domestic trend (the second structural shock) has no long-run effect on terms-of-trade. To identify the transitory shocks we assume that the third element in the fourth column of B_0^{-1} is zero. This implies that that the third structural shock (investment demand shock) has no contemporaneous effect on consumption. Since we are only interested in the impulse responses of the data to the two permanent shocks, this latter restriction is irrelevant.

```
% restrictions.m
function q=restrictions(B0inv)
global GAMMA SIGMA alpha beta alpha_perp beta_perp Xi p
K=size(B0inv,1);
THETA1=Xi*B0inv;
% This is Upsilon F=vec(B0inv*B0inv'-SIGMA(1:K,1:K));
% Long run and short run restrictions
q=[F; B0inv(3,3); THETA1(1,2); THETA1(1,3); THETA1(1,4); THETA1(2,3); THETA1(2,4);
THETA1(3,3); THETA1(3,4); THETA1(4,3); THETA1(4,4)];
q'+1;
```

where the notation is standard.

Impulse Responses and Forecast Error Variance Decomposition

14. Implement the identification scheme using the MATLAB solver. Check that the solver provides a valid identification and compute the variance-covariance matrix of the identified structural shocks. Please, provide the MATLAB code you are using to identify the shocks in the appendix. It must include a description of the restrictions you impose to identify the structural model.

    If you fail computing the B_0^{-1} matrix using the MATLAB solver, please use the ident.p file. This file works as a standard m-file but the coding is concealed and there is no way to convert the p-file into an m-file. Note that the ident.p file is set up to use a closed form solution to compute the B_0^{-1} matrix using a generic identification based on estimates from the VEC model. To use this function, you need to add the following code to your MATLAB m-file and you need to set the rank equal to 2. You can use any number of lags. Note: Make sure that you don't have any ident.m files in the same folder. The same function can be used in a bootstrap. Note also that it will be impossible to interpret the impulse response function and the variance decomposition using economic theory as the identification is incorrect.

```
% Use generic identification
% Input:
% alpha is the K x r adjustment coefficient matrix
% beta is the r x K cointegration vector
% Gamma = [ Gamma(1) Gamma(2) ....  Gamma(p-1)] coefficent matrix
% sigmahat = Sigma_u (the residual covariance matrix)
```

```
% K is the number of variables in the VAR
% p is the number of lags in underlying VAR
% r is the rank
%
% Output:
% invB0 is the inverse of the B0 matrix
% Xi is the C(1) matrix
[invB0,Xi]=ident(alpha,beta,Gamma,sigmahat,K,p,r)
```

Answer: After implementation of the matlab solver we have an estimate of the B_0^{-1}-matrix. The next step is to check that the identification is valid. First of all we need to make sure that the sign of the shocks are the same when implementing a bootstrap procedure as we do below. For example, we may be interested in the effects of positive permanent shocks. In this case we need to check that the first two elements in the diagonal of B_0^{-1} are positive. If this is not the case, then we need to switch the sign of all elements in the same column. Then we need to confirm that the restriction on Υ is implemented and that the last two columns in this matrix are zero. The restriction on B_0^{-1} can also be checked. Lastly, we can compute the structural shocks w_t and check that the variance-covariance matrix of these shocks is an identity matrix. In some cases we need to increase the number of iterations to allow matlab to find a solution.

The code to check the identification:

- Switch signs in columns of B_0^{-1}:

  ```
  % Switch signs if necessary!
  %
  if invB0(1,1)<0
  invB0(:,1)=-invB0(:,1);
  end
  ```

- Check that $\beta\Xi = 0$

  ```
  display(beta*Xi,'beta*Xi should be zero');
  ```

- Check that $\Xi B_0^{-1} = \Upsilon$

  ```
  display(Xi*invB0,'(3) C(1)*B0^1 should be Upsilon zeros(K,r)');
  display(Upsilon,'where Upsilon');
  ```

- Check that the variance-covariance of structural shocks is the identity matrix.

  ```
  display(inv(invB0)*sigmahat*inv(invB0)','(4) Covariance matrix of structural
  shocks w_t should be I_K');
  ```

15. Estimate the structural VAR model and compute impulse response functions (using the standard residual based recursive design bootstrap and IRF confidence bands computed using the delta method) and variance decomposition (with bootstrap standard errors using Efron's percentile intervals). Focus only on the impulse responses of the data to the two permanent shocks. You can show forecast error variance decomposition in either a table or in a graph. Interpret your results.

Answer: Implementing the matlab solver above and compute the impulse responses and the variance decompositions and computing standard errors using bootstrap with 500 trials checking that the identification is valid in each bootstrap replication including whether the simulated VEC model satisfies our assumptions we can show the results in graphs. In these estimations we assume that $b = 0.75$ in the theoretical cointegration vector.

The implied impulse responses to a positive foreign and domestic permanent shock are shown below.

Figure 3: IRF: Positive permanent foreign shock.

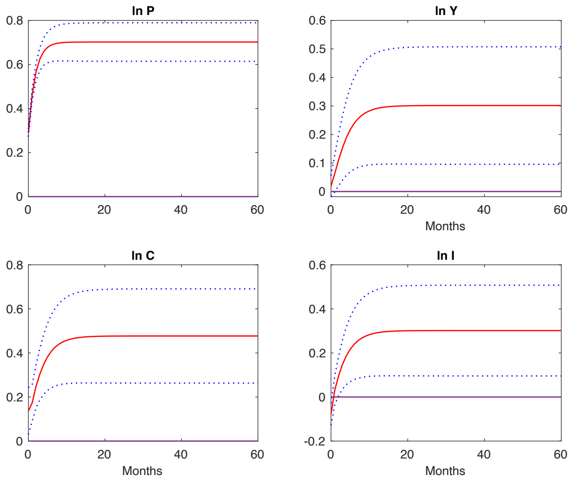
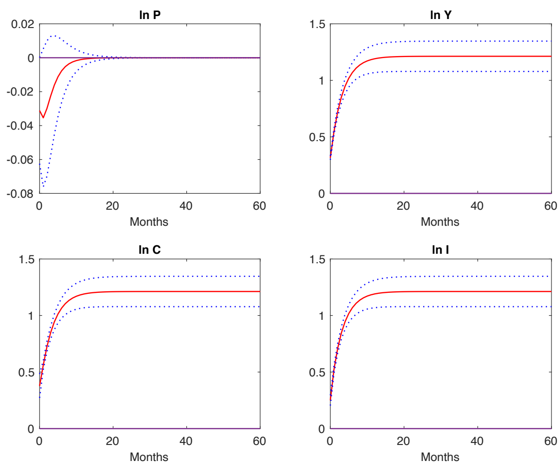


Figure 4: IRF: Positive permanent domestic shock



The impulse responses accords with priors and the implied small open economy restriction is clearly visible.

Variance decompositions are shown below.

Figure 5: FEVD: Positive permanent foreign shock.

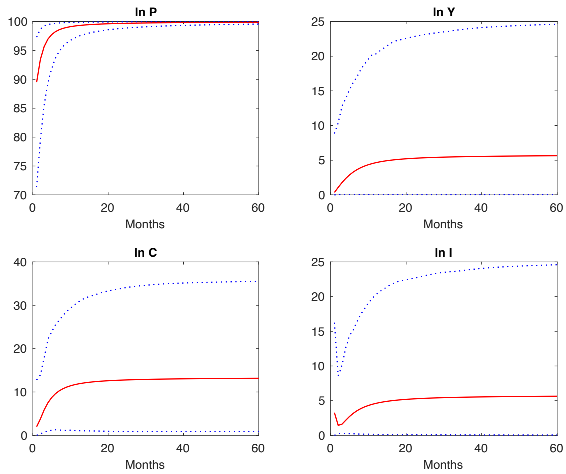
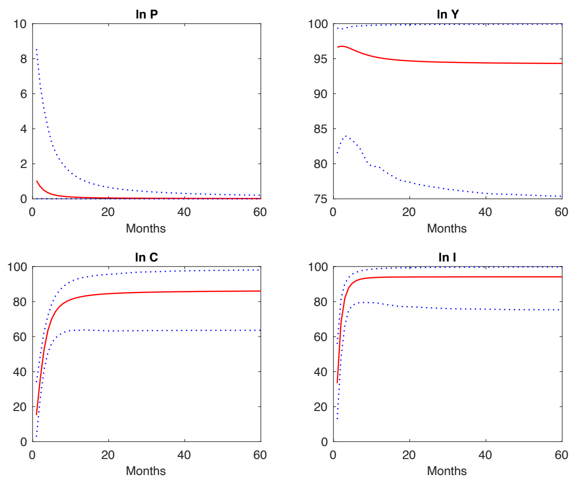


Figure 6: FEVD: Positive permanent domestic shock



A surprising result is that the foreign shock does not explain much of the domestic variables. Results could vary across data sets but the main conclusions should be the same.

16. Instead of implementing the theoretical cointegration vector you can use the estimated cointegration vector still imposing $r = 2$. Identify the structural shocks using long-run restrictions and plot the implied impulse responses of the data together with confidence bands. Focus only on the effects of the permanent shocks. Compare your results to what you previously found using the theoretical cointegration vector.

If you fail computing the B_0^{-1} matrix using the MATLAB solver, please use the ident.p file again. The code is generic and works for any cointegration vector. However, you cannot compare your results in any meaningful way.

Answer:

Figure 7: IRF: Positive permanent foreign shock. Estimated cointegration vector.

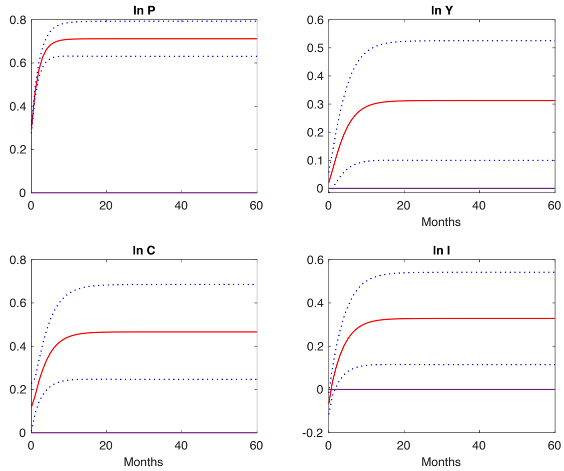
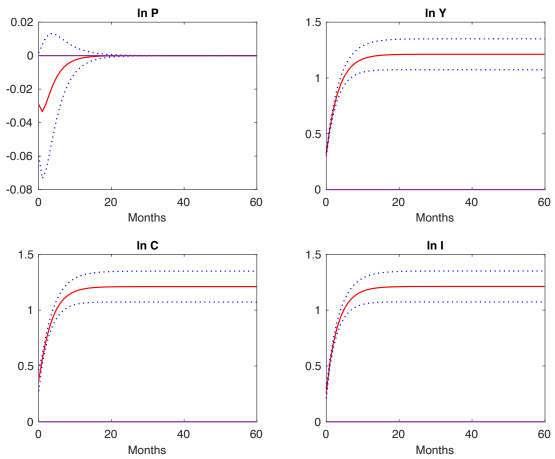


Figure 8: IRF: Positive permanent domestic shock. Estimated cointegration vector.



Comparing with the IRF's and FEVD's using the theoretical cointegration vectors we find only minor differences. The reason for this is that the theoretical and estimated cointegration vectors are very similar. This implies that the estimates of the B_0^{-1} matrix are similar and together with similar estimates of the VEC model, the implied IRF's and FEVD's must also be similar. Note that in some data sets, there will be a minor difference. A good answer should include the estimates of α, β, the companion matrix

A and the estimated B_0^{-1} matrix.

Figure 9: FEVD: Positive permanent foreign shock. Estimated cointegration vector.

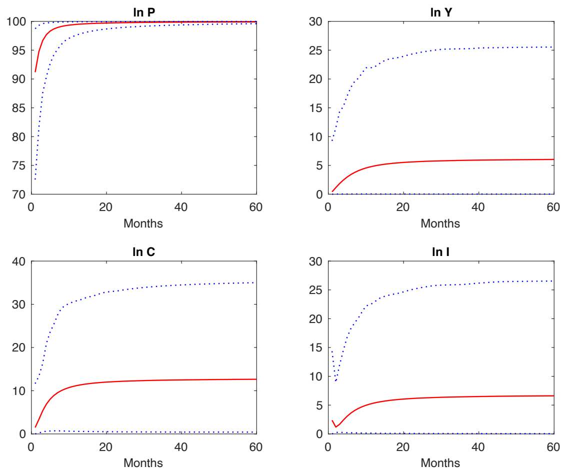
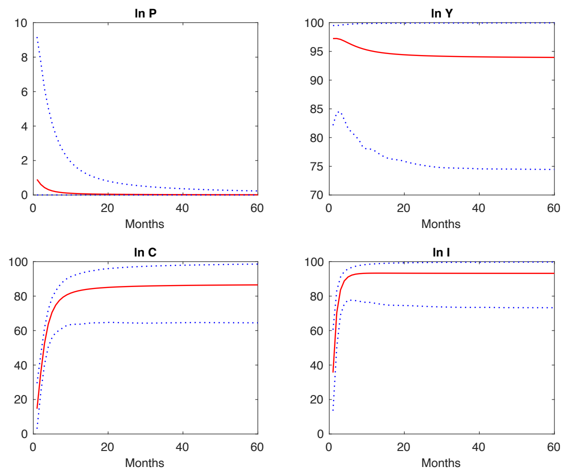


Figure 10: FEVD: Positive permanent domestic shock. Estimated cointegration vector.



Extensions

17. An alternative to using the MATLAB solver to compute the B_0^{-1} matrix is to use the approach suggested by Warne (1993). Outline this approach and show how the Warne approach can be used to identify the structural shocks in your preferred VEC model. Discuss both the identification of permanent and transitory shocks.

Answer: The underlying idea of this approach is to first decompose

$$B_0^{-1} = \left[\begin{array}{c} F_k \\ F_r \end{array} \right]$$

and then we compute the two parts independently since the identification of permanent shocks are independent on the identification of the transitory shocks. Consider first the identification of the permanent shocks. Following Warne we first need to define the matrix Υ_0 such that $\beta'\Upsilon_0 = 0$. We can use iether the theoretical or the estimated cointegration vector to solve for Υ_0. Given a Cholesky decomposition of

$$\left(\Upsilon_0'\Upsilon_0\right)^{-1}\Upsilon_0'\Xi\Sigma_u\Xi'\Upsilon_0\left(\Upsilon_0'\Upsilon_0\right)^{-1}$$

where we note that given our choice of Υ_0 and the estimated VEC model the expression above is known. The Cholesky decomposition of this matrix is denoted π. This allows us to compute $\Upsilon = \Upsilon_0\pi$ in the common trend model. The permanent shocks are then identified, $F_k = \left(\Upsilon'\Upsilon\right)^{-1}\Upsilon'\Xi$.

To identify the transitory shocks, we first define a matrix U

$$U = \left[\begin{array}{cc} 0_{r\times K-r} & I_r^+ \end{array} \right]$$

defining $\xi = \alpha\left(U\alpha\right)^{-1}$ we then computea matrix Q which is the Cholesky decomposition of $\xi'\Sigma_u^{-1}\xi$ and then finally we identify the transitory shocks using the matrix $F_r = Q^{-1}\xi'\Sigma_u^{-1}$

In our example, we have two theoretical cointegration vectors. If we define

$$\Upsilon_0 = \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \\ 1-b & 1 \\ 0 & 1 \end{array} \right]$$

and given that π is a lower triangular matrix we find that

$$\left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \\ 1-b & 1 \\ 0 & 1 \end{array} \right] \left[\begin{array}{cc} \pi_{11} & 0 \\ \pi_{21} & \pi_{22} \end{array} \right] = \left[\begin{array}{cc} \pi_{11} & 0 \\ \pi_{21} & \pi_{22} \\ (1-b)\pi_{11} + \pi_{21} & \pi_{22} \\ \pi_{21} & \pi_{22} \end{array} \right]$$

where we see that the identifying restriction that the second permanent shock has no long-run effect on the first variable is imposed.

Since we have two transitory shocks and to implement the restriction that the third element in the third column is equal to zero we define

$$U = \left[\begin{array}{cccc} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

It is now straightforward to code this. For this ordering of the variables we can verify that the B_0^{-1} matrix found using the Warne method is equal to the one found by the solver. It could be that we need to switch signs on one or two columns of the solver solution, but the absolute values are identical.

18. Code this identification and compute the implied B_0^{-1} matrix and show that it is identical to the one found by the solver. Please, provide the code you are using in the appendix.

Answer: The following code implements this identification. The notation is standard.:

```
function [invB0,Xi]=ident2(alpha,beta,Gamma,sigmahat,K,p,r)
% Use the matlab function null to compute orthogonal complements
beta_perp=null(beta);
alpha_perp=null(alpha');
% Compute GammaSum
GammaSum=Gamma(1:K,1:K);
if p>2
for i=1:p-2
GammaSum=GammaSum+Gamma(1:K,i*K+1:i*K+K);
end;
end
% Compute Xi=C(1)
Xi=beta_perp*inv(alpha_perp'*(eye(K)-GammaSum)*beta_perp)*alpha_perp';
% Identification of permanent shocks and transitory shocks
% according to Warne.
% Impose restrictions to identify permanent shock
% Upsilon_0
u31 = beta(2,1);
u32 = 1;
u41 = 0;
u42 = 1;
%Upsilon0 = [1 0;0 1; u31 u32;u41 u42];
% Impose restrictions to identify transitory shocks
% Impose restriction that B0^{-1}_{2,2} = 0
% Code below use Umat = TID
% If you want to use an automatic selection,
% set TID=0.
TID = 0;
MHLP=inv(Upsilon0'*Upsilon0)*Upsilon0'*Xi;
pipit=MHLP*sigmahat*MHLP';
pimat=chol(pipit)';
Upsilon=Upsilon0*pimat;
Fk=inv(Upsilon'*Upsilon)*Upsilon'*Xi;
%display(Fk,'Fk matrix');
% Identification transitory shocks
Umat=zeros(r,K);
% If TID=0, use automatic Umat, otherwise use TID defined above
if TID==0;
i=1;
while i<=r;
Umat(i,K-i+1)=1;
```

```
i=i+1;
end;
else
Umat = TID;
end
xi=alpha*inv(Umat*alpha);
i=1;
while i<=K;
j=1;
while j<=r;
if abs(xi(i,j))<=1E-12; % just to make sure that elements are = 0
xi(i,j)=0;
else
end
j=j+1;
end
i=i+1;
end
qr=chol(xi'*inv(sigmahat)*xi)';
Fr=inv(qr)*xi'*inv(sigmahat);
%display(Fr,'Fr matrix');
% Putting it all together to compute B0inv
invB0 = inv([Fk;Fr]);
end
```

Using the code above we find that the following estimate of B_0^{-1}

$$\begin{array}{cccc} 0.28952450935 & -0.03118291330 & -0.09419711329 & -0.00258368709 \\ 0.01885033074 & 0.31412528439 & -0.05192604615 & 0.01910139425 \\ 0.13647766008 & 0.37699777451 & 0 & -0.88000270177 \\ -0.07791414260 & 0.24901615613 & -0.34131942386 & 0.02949758117 \end{array}$$

that can be compared to what we find using the matlab solver

$$\begin{array}{cccc} 0.28952450935 & -0.03118291330 & -0.09419711329 & -0.00258368709 \\ 0.01885033074 & 0.31412528439 & -0.05192604615 & 0.01910139425 \\ 0.13647766008 & 0.37699777451 & 0 & -0.88000270177 \\ -0.07791414260 & 0.24901615613 & -0.34131942386 & 0.02949758117 \end{array}$$

Note that there could be numerical differences at 12th decimal point but the first 10/11 decimals must be identical.